4. Optics review

4.1 Ray Optics

Light travels in different optical media in accordance with a set of geometrical rules

4.2 Classical (Wave) Description

Light is an EM wave

4.3 Quantum (Particle) Description

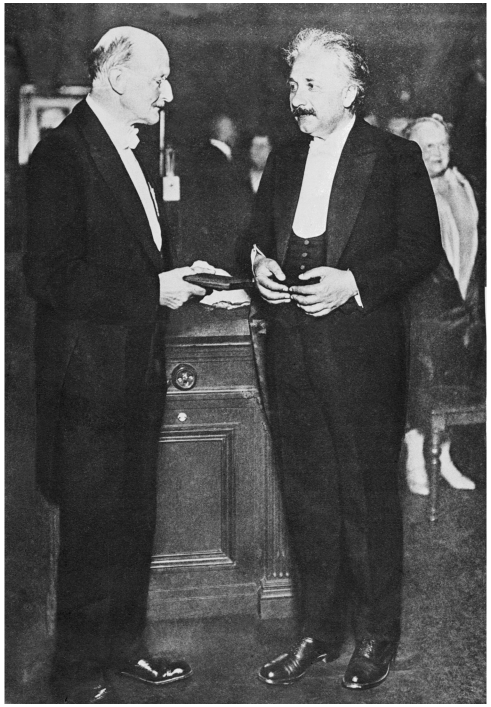
Localized, massless quanta of energy – photons

4.3 Quantum Description of Light

1. Historical perspective

- **Max Planck (1858-1947)**
  1900: Black-body radiation law
  Central assumption:
  Electromagnetic energy can only be emitted in quantized form ("quanta"), i.e. energy can only be a multiple of an elementary unit $E=h\nu$ with h= "Planck" constant, ν= frequency

- **Albert Einstein (1879-1955)**
  1905: Proof of the particle-like behavior of light cames from the photoelectric effect experiment

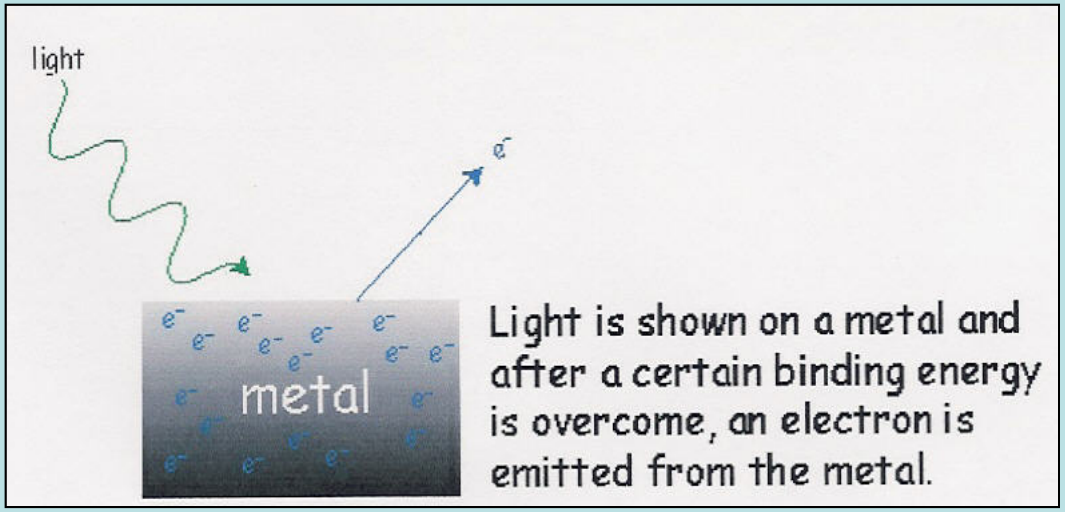


Max Planck presents **Albert Einstein** with the Max-Planck medal, 1929

Quantum Description of Light

2. The Photoelectric effect

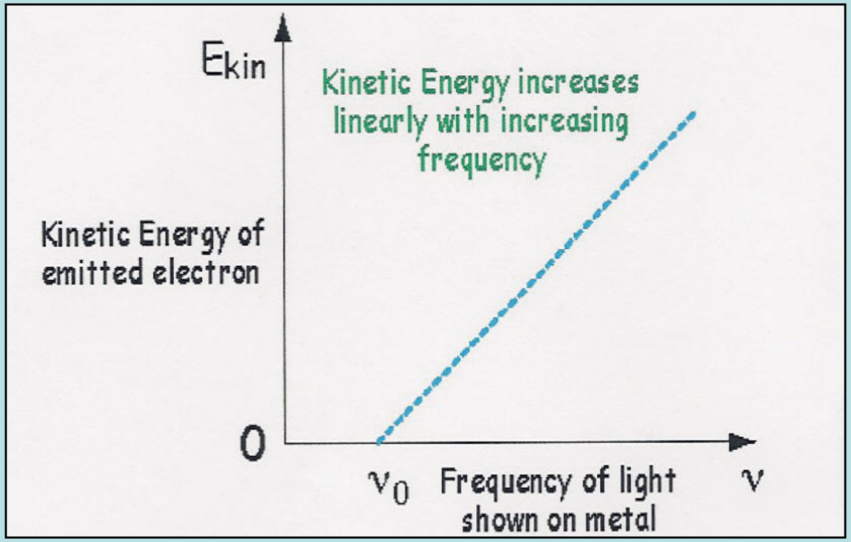
- *1888 Hallwachs and Hertz:*
  Irradiation of negatively charged metal plate with UV light
  => emission of electrons leading electrical discharge
- *Most important feature of the photoelectric effect:*
  an electron is emitted from the metal with a specific kinetic energy (speed)



Quantum Description of Light

2. The Photoelectric effect - Observations

- Kinetic energy do not change with light intensity but changes with the frequency of the incident light
- there is a critical frequency below which no electrons are emitted
- Slope of the line is Planck's constant



Quantum Description of Light

2. Photoelectric effect – Interpretation

- 1905 by A. Einstein (Nobelprize 1921)
- We can write an equation for the kinetic energy of the emitted electron, where **h** is the Planck constant:

Kinetic energy of the electron emitted from the metal ➡ $E_{kin} = h\nu - h\nu_0$

Energy of the photon ($h\nu$)

Energy needed to **eject** an electron from the metal ("work function of a metal") ($h\nu_0$)

Quantum Description of Light

Photon Properties:

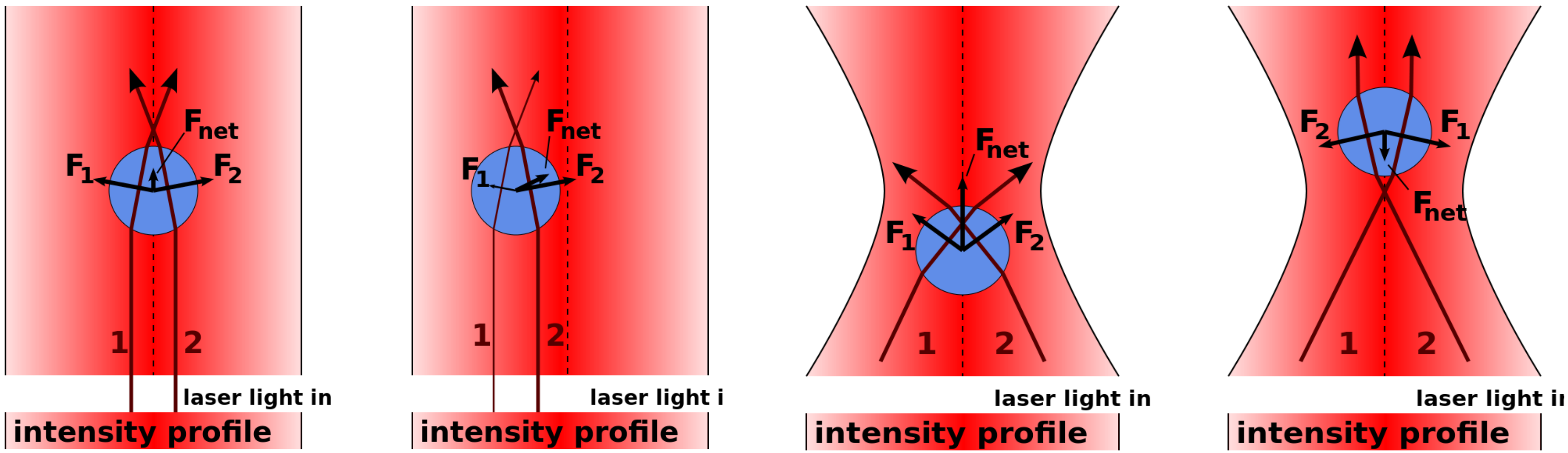
- Energy $E_{ph} = h \cdot \nu = \dfrac{h \cdot c}{\lambda}$
- Momentum: $p_{ph} = \dfrac{E}{c} = \dfrac{h}{\lambda}$

  Derives from:

  $$E^2 = p^2c^2 + m^2c^4.$$
- Rest mass: $m_{0,ph} = 0$

with h = 6.6261965 x 10^{-34} Js, c = 2.9979250 x 10^{8} m/s

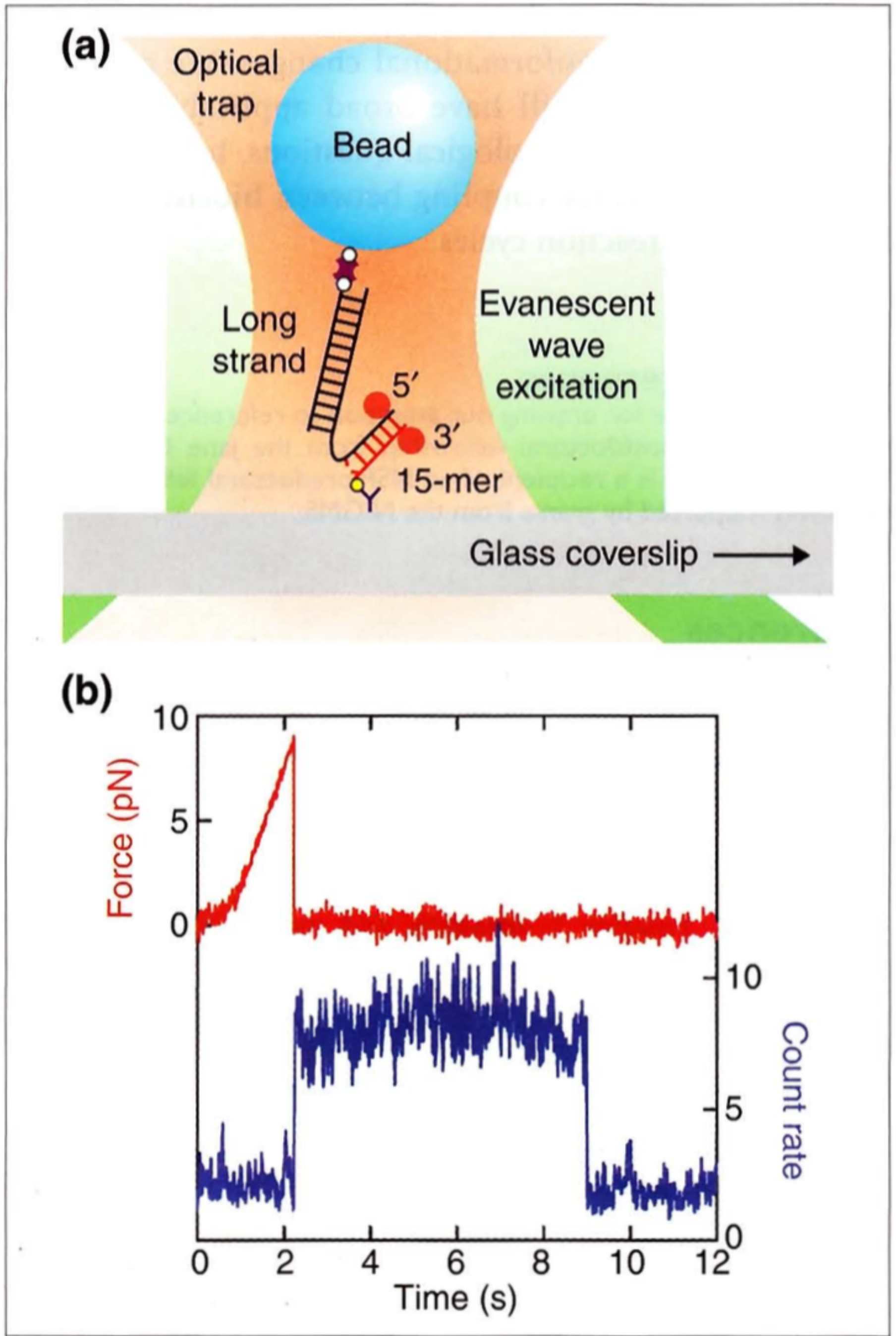
Optical tweezers



Source: R. Koebler

Lateral positioning

Longitudinal positioning



Review

A combined optical trapping and fluorescence experiment to unzip DNA. **(a)** A cartoon of the simplified experimental geometry (not to scale). A bead was tethered by a digoxygenin-based linkage (blue and yellow) to the coverglass surface through a DNA molecule, consisting of a long segment (black) joined to a shorter 15 base-pair strand that forms a duplex region (red). The bead (blue) was captured by the optical trap and force was applied to unzip the short duplex. Tetramethylrhodamine (TAMRA) dyes attached at the ends of the DNA strands provide a fluorescence signal (red dots). **(b)** Simultaneous records of force (red trace) and fluorescence, measured as the photon count rate (blue trace). Rupture occurred at $t \approx 2$ sec at an unzipping force of 9 pN. The dye unquenched at the point of rupture, and later bleached at $t \approx 9$ sec. See text for further details.

Research article

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Combined optical trapping and single-molecule fluorescence

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Wave/Particle Duality - Light

Photons versus EM waves

Apart its **wave-like properties**, that are demonstrated by:

- Refraction
- Interference and
- Polarization

Light also shows **particle-like behavior**

- « Low intensity » emission and absorption (Shot noise)
- Photoelectric effect

Wave / Particle Duality

High frequency (X- or γ-rays)

Momentum and energy of photon increase

Photon (particle) description dominates

Low Frequency (radio waves)

Interference/diffraction easily observable

Wave description dominates

What about matter ?

Our understanding of the structure of matter was led by a series of breakthrough in early 20th century.

Matter

Particle behavior

Kinetic energy $E_{kin} = \frac{1}{2} m v^2$

Momentum $p = mv$

Translational energy described
by the classical Newtonian mechanics

Wave-like behavior

Wavelength
(de Broglie wave) $\lambda = \frac{h}{m\nu}$

h = Planck constant

Quantized energy obtained
by solution of the Schrödinger equation

Wave Nature of Electron

Theory of light 1920's:

"Classical" wave theory (since 1690):
light is a wave phenomenon

Photoelectric effect:
light has particle properties

Einstein: Relativity

Louis de Broglie

Born: 15 Aug 1892 in Dieppe, France
Died: 19 March 1987 in Paris, France

Louis de Broglie (1924):

Might electrons and other "particles" exhibit wave-like properties?

Wave Nature of Electron

The de Broglie Hypothesis (1924):

Relativity

$$E = mc^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

Kinetic energy term (p^2c^2)

Rest mass energy term ($m_0^2c^4$)

rest mass $m_{0,ph}=0$

Momentum of a photon

$$p = \frac{E}{c}$$

Photoelectric effect

$$E = h\nu = \frac{hc}{\lambda}$$

$$\frac{h}{\lambda} = \frac{E}{c}$$

Wavelength energy relation

$$\lambda = \frac{h}{p}$$

for photon

The de Broglie Hypothesis

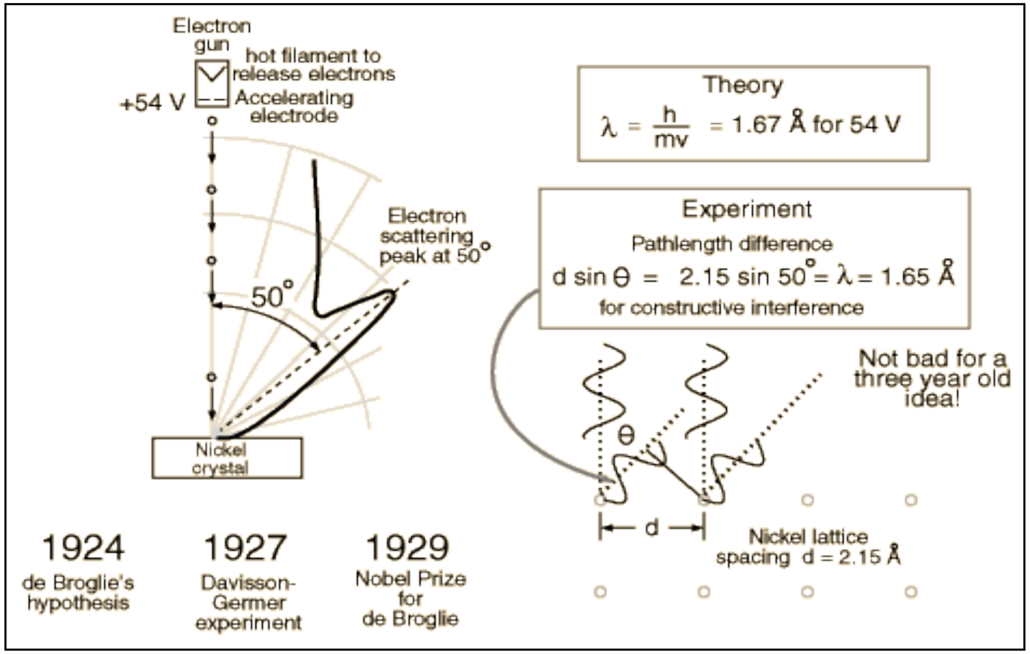
$$\lambda = \frac{h}{p} \overset{?}{=} \frac{h}{mv}$$

for an electron?

Wave Nature of Electron

Confirmation of de Broglie's Hypothesis:

- **1927 Clinton Davisson and Lester Germer: Davisson-Germer experiment**



- Slow electrons impact on a crytalline Nickle target

➡ Angular distribution of reflected electron intensity = angular distribution predicted for X-rays (waves) by Bragg(*).

(*) *Friedrich and Knipping experimentally showed the interference of X-rays (1912); the results were interpreted by Bragg*

Wave and Particle Concepts

De Broglie's equations for particles

Momentum $p = mv = \frac{h}{\lambda}$

de Broglie Wavelength $\lambda = \frac{h}{p}$

Frequency $\nu = \frac{E}{h}$
(obeys Einstein relation)

Each equation contains

- **particle concepts (p and E)**
  AND
- **wave concepts (λ and ν).**

Wave Mechanics

The wave function and the wave equation

- **De Broglie waves** are represented by a simple quantity called a **wave function Ψ**
- in quantum mechanics a particle is completely described by the **wave function**, which is a complex function of time and position **$\Psi=\Psi(x,t)$**
- The **wave function** can be determined by solving the **Schrödinger** wave equation

Schrödinger's wave equation

Significance of terms

Potential energy

$$\frac{-h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2}+(V-E)\psi=0$$

Allowed System Energy
(potential and kinetic energy)

- $\psi^2(x)dx$ describes the probability of finding the particle in the length segment between x and $x + dx$.

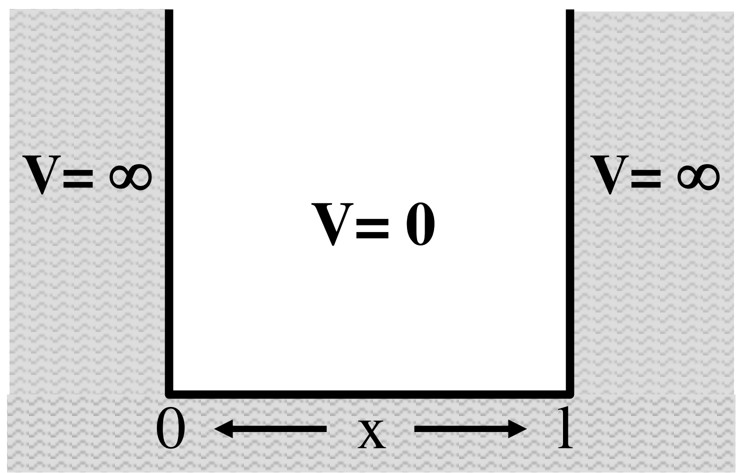
Particle in a Box

Experiment consisting of a single particle bouncing around inside a box with the following conditions

- single point particle
- no forces
  (zero potential energy V(x))
- at the walls of the box:
  potential rises to infinity
  (impenetrable wall)

Let's solve:

$$\frac{-h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2}+(V-E)\psi=0$$



Characteristics of the potential V(x)

Particle in a Box

For a particle confined to moving along the x-axis :

$$\frac{h^2}{8\pi^2 m}\frac{\partial^2\Psi}{\partial x^2}+(E-V)\Psi=0 \quad \rightarrow \quad -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=E\psi$$

Boundary condition:

$$V(x)=0$$
$$\psi(x)=0 \quad \text{at } x=0 \text{ and } x=l$$

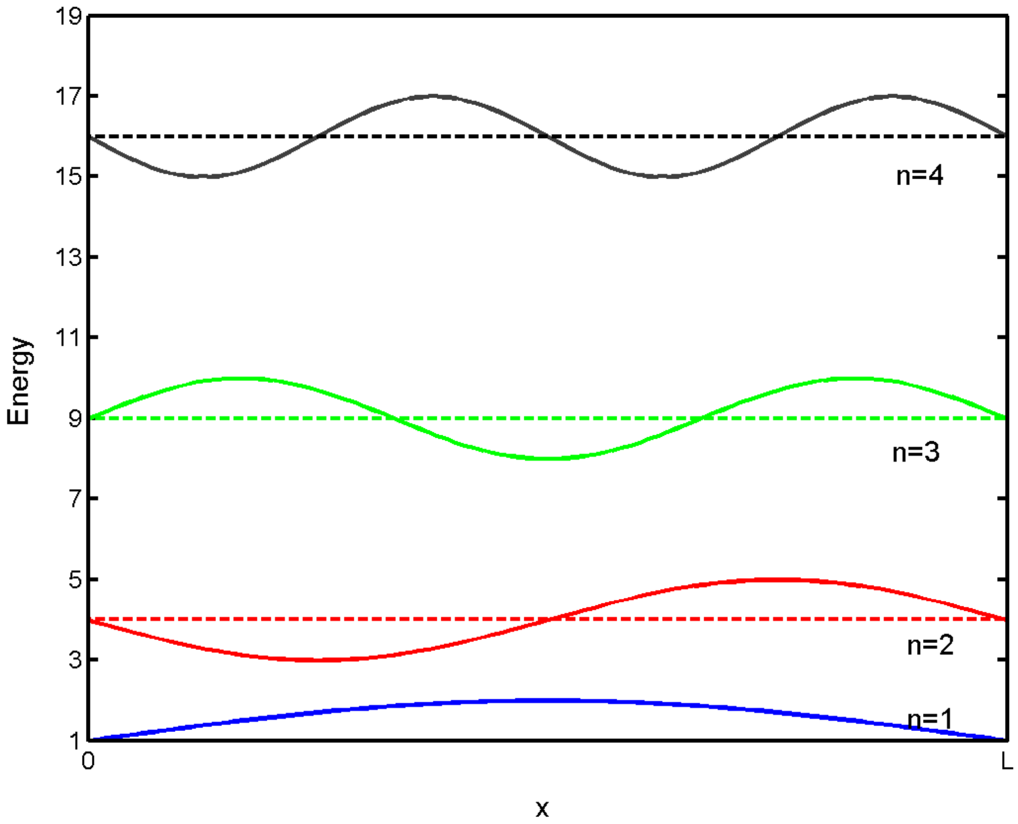
The quantized energy levels can be determined by solving the equation:

$$E_n=\frac{n^2h^2}{8ml^2} \qquad \psi_n(x)=\left(\frac{2}{l}\right)^{1/2}\sin\left(\frac{n\pi x}{l}\right)$$

Eigen values: E_n
Eigen functions: Ψ_n

n = positive integer

Particle in a box



- In general the n^{th} wavefunction has (n-1) nodes.
- Energy levels

$$E_n = \frac{n^2 h^2}{8ml^2}$$

(node = place where the wavefunction is zero)

Particle in a Box

The gap between two successive levels E_n and E_{n+1} can be given as

$$E_n = \frac{n^2 h^2}{8ml^2} \rightarrow \Delta E = (2n+1)\frac{h^2}{8ml^2}$$

This equation reveals that the gap between two successive levels decreases as l^2 when the length of the box increases. Translational energies of atoms and molecules, which involve displacement over a large distance compared to the atomic scale, will have very small spacing and can be considered not to be quantized—that is, treatable by classical mechanics. This model also explains that when a bond is formed between two atoms, the length in which the bonding electrons are confined increases (spreads over two atoms). Consequently, the energy E_n is lowered, stabilizing the formation of the bond, because a lower energy configuration is always preferred.